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Radiative symmetry breaking and dynamical origin of the cosmological constant in ϕ^4 theory with a non-linear curvature coupling

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Abstract

A scalar self-interacting theory non-linearly coupled with some power of the curvature has the possibility of explaining the current smallness of the cosmological constant. In Inagaki *et al* (2005 *J. Cosmol. Astropart. Phys.* JCAP06(2005)010) a symmetry property of the model has been studied in arbitrary dimensions and a solution of the cosmological constant problem has been found in four dimensions. Here one concentrates on a massless scalar field in the four-dimensional Friedmann–Robertson–Walker spacetime with flat spatial part. One applies the same analysis as in Inagaki *et al* (2005 *J. Cosmol. Astropart. Phys.* JCAP06(2005)010) and shows the phase structure of radiative symmetry breaking. One also reviews a dynamical resolution of the cosmological constant problem.

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1. Introduction

One of the most important and mysterious problems in the present cosmology is the origin of dark energy. It has an anti-gravity contribution to accelerate the universe expansion. Even in particle physics, much attention has been paid to the problem and new field theoretical models are considered to explain the acceleration of our universe. A simple candidate for the dark energy is found in the vacuum energy or cosmological constant. However, the scale of the dark energy is about 120 orders of magnitude smaller than the Planck scale. We cannot avoid fine-tuning to obtain a suitable scale vacuum energy in most particle physics models.

An interesting possibility of solving the dark energy problem is found in a modification of the gravity–matter coupling. Nojiri and Odintsov have proposed the class of dark energy models where the dark energy field couples with some power of the curvature [2] (see also a related model in [3]). The model naturally resolves the problem of dark energy dominance in the current universe. In [1] the symmetry properties of such models have been studied with the example of scalar self-interacting theory with a non-linear curvature coupling in arbitrary dimensions. It is found that the phase structure of the model strongly depends on the curvature power in 3.8 spacetime dimensions.

In the present paper, we focus on the four-dimensional limit of the theory and continue the study of scalar self-interacting theory with a non-linear curvature coupling. It should be noted that the theory is not a standard one, in the sense that it is not multiplicatively renormalizable in curved spacetime [4]. We regard it as an effective theory stemming from a more fundamental theory at Planck scale. In our analysis we neglect the quantum gravity effects. We also assume that the spacetime curved slowly and neglect the higher order terms about the curvature. It is appropriate for the study of quantum effects in the inflationary universe and also in the late-time, dark energy universe.

First, we introduce the scalar theory non-linearly coupled with some power of the curvature. To find the properties at the weak curvature limit, we apply the Riemann normal coordinate expansion and evaluate the effective Lagrangian. The one-loop effective Lagrangian is obtained in close analogy with multiplicatively renormalizable theories. Next, we consider the massless scalar field and investigate radiative symmetry breaking which is caused by the loop corrections of the scalar field. In our theory, the expectation value of the scalar field corresponds to an order parameter of radiative symmetry breaking. We evaluate the effective Lagrangian numerically in four dimensions and find the phase structure of the theory in four dimensions. Finally, we show the solution of the Einstein equation and discuss dynamical resolution of the cosmological constant problem.

2. ϕ^4 theory with a non-linear curvature coupling

As is well known, ϕ^4 theory is one of the simplest models where spontaneous symmetry breaking takes place. It is more instructive to consider the ϕ^4 theory as a prototype model to study the influence of a non-linear curvature coupling. Here we extend the ϕ^4 theory to include a coupling with some power of the curvature,

$$S = \int \mathrm{d}^D x \sqrt{-g} \left[\frac{1}{2\kappa^2} R + \left(\frac{R}{M^2} \right)^{\alpha} \mathcal{L}_d \right]. \tag{1}$$

where M is an arbitrary mass scale, g is the determinant of the metric tensor $g_{\mu\nu}$, and $\mathcal{L}_d$ is the ordinary Lagrangian density of the ϕ^4 theory,

$$\mathcal{L}_d(\phi_0) = \frac{1}{2}\phi_{0;\mu}\phi_0{}^{;\mu} - \frac{\xi_0 R}{2}\phi_0^2 + \frac{\mu_0^2}{2}\phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4, \tag{2}$$

where ϕ_0 is a real scalar field. Our sign convention for the metric is $(+ - - - \cdots)$.

The action (1) is invariant under the discrete transformation, $\phi \to -\phi$. A non-vanishing expectation value for the field ϕ breaks this discrete Z_2 symmetry spontaneously. The expectation value for ϕ is found by observing the stationary point of the effective action $\Gamma[\phi_c]$.

In a constant curvature spacetime, the non-linear curvature coupling disappears by the transformation

$$\phi \to \left(\frac{R}{M^2} \right)^{-\alpha/2} \phi, \qquad \lambda_0 \to \left(\frac{R}{M^2} \right)^{\alpha} \lambda_0. \tag{3}$$

Hence a non-trivial effect of the non-linear curvature coupling will be found only in a non-static and/or inhomogeneous spacetime.

Here we adopt the Riemann normal coordinate expansion [5–7] and evaluate the one-loop effective Lagrangian. It is divergent in four dimensions. To obtain the finite result we have to renormalize the theory. Here we impose the following renormalization conditions,

$$\left.\frac{\partial^2\Gamma[\phi]}{\partial\phi^2}\right|_{\phi=0} \equiv \left(\frac{R}{M^2}\right)^{\alpha}\left(\mu_r^2-\xi_r R\right), \quad \left.\frac{\partial^4\Gamma[\phi]}{\partial\phi^4}\right|_{\phi=M} \equiv -\left(\frac{R}{M^2}\right)^{\alpha}\lambda_r, \tag{4}$$

where M is the renormalization scale. From these conditions one obtains the renormalized parameters μ_r, ξ_r and λ_r. By using these renormalized parameters the effective Lagrangian density reads at the four-dimensional limit

$$\begin{aligned}
\mathcal{L}_{\rm eff}^{\rm 4D} &= \frac{1}{2\kappa^2}R + \left(\frac{R}{M^2}\right)^{\alpha}\left(\frac{1}{2}\phi_{;\mu}\phi^{;\mu} - \frac{\xi_r R}{2}\phi^2 + \frac{\mu_r^2}{2}\phi^2 - \frac{\lambda_r}{4!}\phi^4\right)\\
&\quad + \frac{\hbar}{128\pi^2}\left[\lambda_r^2\phi^4\left(\frac{\lambda_r M^2}{\chi^2(M^2)} - \frac{1}{6}\frac{\lambda_r^2 M^4}{\chi^2(M^2)}\right) + \lambda_r\phi^2\chi^2(0)\right.\\
&\quad + \frac{3}{4}\lambda_r^2\phi^4 - 2(\chi^2(\phi))^2\ln\frac{\chi^2(\phi^2)}{\chi^2(0)} - \frac{1}{2}\lambda_r^2\phi^4\ln\frac{\chi^2(0)}{\chi^2(M^2)}\\
&\quad - \left\{\frac{\lambda_r^2}{4}\phi^4\left(\frac{1}{\chi^2(M^2)} - \frac{2\lambda_r M^2}{(\chi^2(M^2))^2} + \frac{2}{3}\frac{\lambda_r^2 M^4}{(\chi^2(M^2))^3}\right)\right.\\
&\quad \left.\left. + \lambda_r\phi^2 - 2\chi^2(\phi^2)\ln\frac{\chi^2(\phi^2)}{\chi^2(0)}\right\}\mathrm{Tr}\,\xi\right],
\end{aligned} \tag{5}$$

with

$$\mathrm{Tr}\,\xi = \alpha\left(\frac{\Box R}{R} - \frac{R^{;\mu}R_{;\mu}}{R^2}\right), \tag{6}$$

$$\chi^2(\phi^2) = -\mu_0^2 + \frac{\lambda_0}{2}\phi^2 + \left(\xi_0 - \frac{1}{6}\right)R - \frac{\alpha}{2}\left(\frac{\alpha}{2}-1\right)\frac{R^{;\mu}R_{;\mu}}{R^2} - \frac{\alpha}{2}\frac{\Box R}{R}. \tag{7}$$

Therefore the ultraviolet divergences disappear after the one-loop renormalization. Of course, the question of dependence from the regularization in such effective models remains at higher order.

3. Radiative symmetry breaking in four-dimensional FRW spacetime

It is expected that the non-linear curvature coupling in our model leads to non-trivial consequences for the phase structure in a non-static spacetime. It is more interesting to consider the radiative symmetry breaking at $\mu_r = 0$. In this case, spontaneous symmetry breaking cannot take place on the classical level of the theory. It is expected that the radiative correction plays an essential role and the curvature effect is more important for symmetry properties.

Here we consider the model in the spatially flat FRW spacetime in four dimensions. It is defined by the metric

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)[\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_2]. \tag{8}$$

The time dependence of the scale factor is assumed to be $a(t) = a_0 t^{h_0}$. All mass scales are normalized by an arbitrary mass scale M and $\hbar = 1$.

First we consider the stationary and spatially homogeneous ϕ. In this case the kinetic term of ϕ disappears. One can define the effective potential by the opposite sign of the effective Lagrangian, $V(\phi) \equiv -\mathcal{L}_{\rm eff}^{\rm 4D}$. The expectation value of the field ϕ is determined by observing

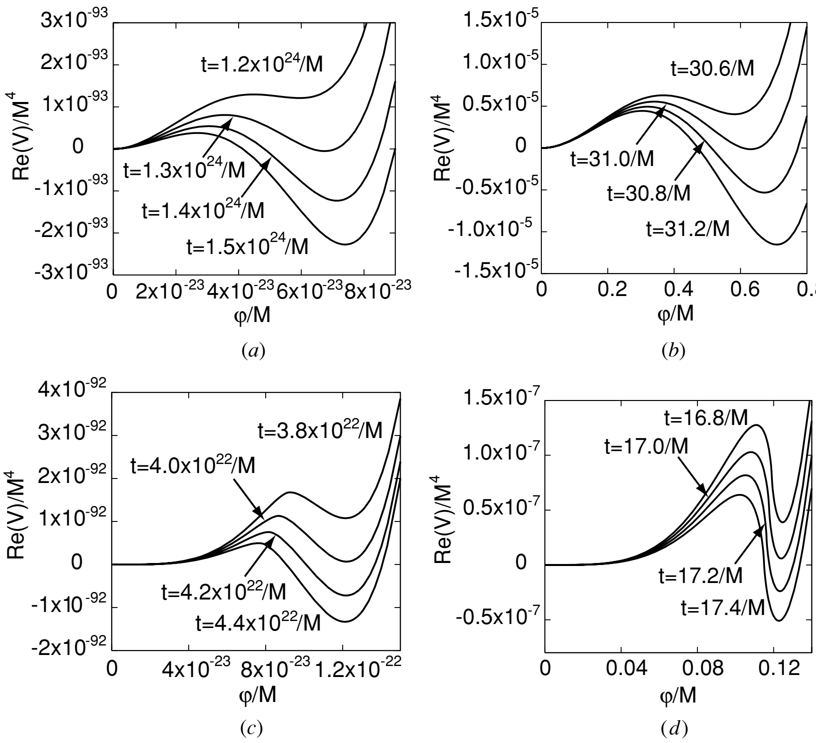

Figure 1. Behaviour of the effective potential for $h_0 = 2$, $\mu_r = 0$, $\lambda = 1$ and $D = 4$. (*a*) $\alpha = 0$, $\xi = \xi_{\rm conformal}$, (*b*) $\alpha = 1$, $\xi = \xi_{\rm conformal}$, (*c*) $\alpha = 0$, $\xi = 0$ and (*d*) $\alpha = 1$, $\xi = 0$.

the minimum of the effective potential. The effective potential develops a small imaginary part for a negative $\chi(\phi)$. Below, we evaluate a real part of the effective potential to find the ground state.

In figure 1 we illustrate the behaviour of the effective potential for a conformal and a minimal gravitational coupling $\xi = 1/6$ and $\xi = 0$ respectively. It should be noted that the theory is conformally invariant only when $\alpha = 0$. Behaviours of the minimal of the effective potential are drawn in figures 2 and 3 with varying λ and h_0. First-order phase transition is observed in figures 2 and 3. However, the radiative correction has only a small effect in an ordinary theory. The expectation value generated by the radiative symmetry breaking is extremely small at $\alpha = 0$, as shown in figure 2. For a positive α the non-linear curvature coupling enhances radiative corrections as curvature decreases. Thus the expectation value $\langle\phi\rangle$ becomes extremely larger in comparison with that for the case $\alpha = 0$. In figure 3 we observe two steps of the transition at $h_0 = 1.2$. In the case of negative α the radiative correction is suppressed as the curvature decreases and then radiative symmetry breaking does not occur.

As shown in figures 2 and 3, the expectation value $\langle\phi\rangle$ has non-negligible time dependence, especially for $\alpha = 1$. Hence we next consider the spatially homogeneous but time-dependent ϕ at $\alpha = 1$. The time evolution of $\langle\phi\rangle$ is described by the equation of motion:

$$\frac{1}{\sqrt{-g}}\frac{\delta S}{\delta\phi} = -\frac{\partial V(\phi)}{\partial\phi} - \left(\frac{R}{M^2}\right)^{\alpha}\left[\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{2\alpha}{t}\dot{\phi}\right] = 0. \tag{9}$$

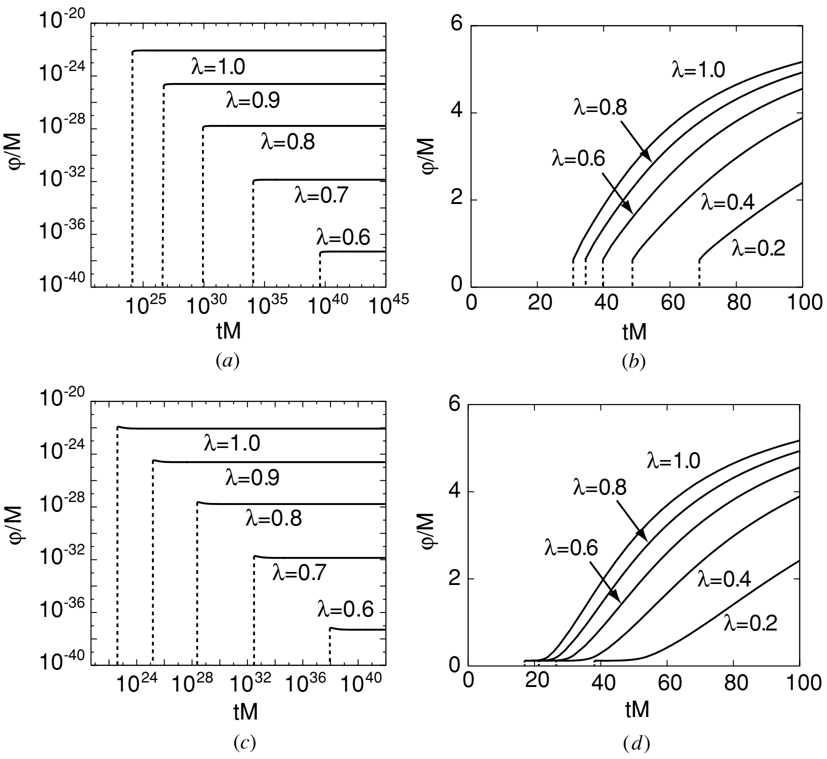

Figure 2. Behaviour of the mass gap for $h_0 = 2$, $\mu_r = 0$ and $D = 4$. (*a*) $\alpha = 0$, $\xi = \xi_{\text{conformal}}$, (*b*) $\alpha = 1$, $\xi = \xi_{\text{conformal}}$, (*c*) $\alpha = 0$, $\xi = 0$ and (*d*) $\alpha = 1$, $\xi = 0$.

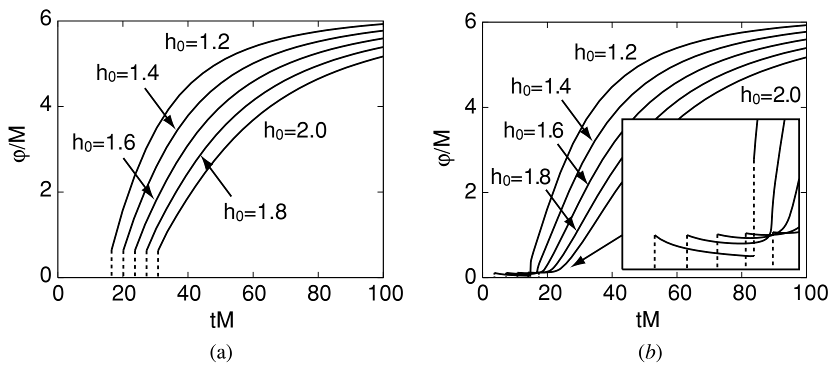

Figure 3. Behaviour of the mass gap for $\alpha = 1$, $\mu_r = 0$, $\lambda = 1$ and $D = 4$. (*a*) $\xi = \xi_{\text{conformal}}$ and (*b*) $\xi = 0$.

To find an exact solution, one needs to solve this equation for a general time-dependent form of ϕ. However, it is instructive to consider the solution of equation (9) for a special form of ϕ. In the present paper, it is assumed that

$$\phi(t) = \langle\phi(t)\rangle = vt^x, \qquad \phi^{;\mu}\phi_{;\mu} = \frac{x^2}{t^2}\langle\phi(t)\rangle^2, \tag{10}$$

where v is a constant parameter. Here we fix the parameter x at $1/2$ or 1 and numerically solve the equation of motion (9).

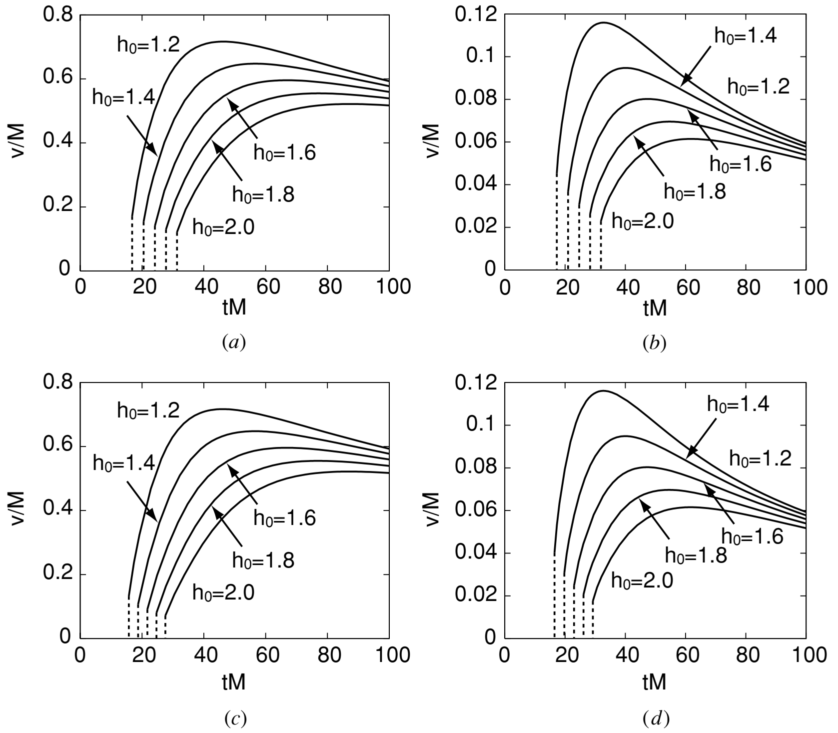

Figure 4. Behaviour of the mass scale v for $\alpha = 1$, $\mu_r = 0$, $\lambda = 1$ and $D = 4$. (*a*) $\langle\phi\rangle = v\sqrt{t}, \xi = \xi_{\mathrm{conformal}}$, (*b*) $\langle\phi\rangle = vt, \xi = \xi_{\mathrm{conformal}}$, (*c*) $\langle\phi\rangle = v\sqrt{t}, \xi = 0$ and (*d*) $\langle\phi\rangle = vt$, $\xi = 0$.

The solution is shown in figure 4. As is seen in figure 4, we observe the first-order phase transition again. In this case the mass scale v depends on the time t again. However, we observe that the mass scale v is almost static around $tM \sim 60$ for $x = 1$ and larger t for $x = 1/2$. It is expected that there is a solution with gradually decreasing x after the first-order transition. It is also interesting that the ξ dependence of v is smaller than results for a stationary ϕ.

4. Resolution of the cosmological constant problem

There are some proposals for solving the cosmological constant problem dynamically [8, 9]. One of the possible solutions was pointed out by Mukouyama and Randall [10]. Here we consider the scalar theory non-linearly coupled with the curvature (1) in the four-dimensional FRW metric with flat spatial part. We apply the analysis by Mukouyama and Randall to our model. A solvable case is found at $\alpha = -1$ [1].

As shown in the previous section, the radiative correction is suppressed when the curvature is small for a negative α. The behaviour of the scalar field ϕ is found by solving the Einstein equation and the field equation for ϕ simultaneously. A solution of these equations is given by

$$\begin{aligned} H &= \frac{h_0}{t}, \qquad \phi = \frac{\phi_0}{t}, \qquad (h_0 > 0), \qquad \text{or} \\ H &= \frac{h_0}{t_s - t}, \qquad \phi = \frac{\phi_0}{t_s - t}, \qquad (h_0 < 0). \end{aligned} \tag{11}$$

Substituting (11) into the Einstein equation and the field equation, we find the solution

$$\phi_0^2 = \frac{3}{\kappa^2 \left\{ \frac{8-9h_0}{24\left(-h_0+h_0^2\right)^2} - \frac{(4-7h_0)\xi}{\left(-h_0+2h_0^2\right)h_0} \right\}}, \tag{12}$$

$$\lambda = -6\kappa^2 h_0 \left\{1 - 2\left(1 - 2h_0\right)\xi\right\} \left\{ \frac{8-9h_0}{24\left(-h_0+h_0^2\right)^2} - \frac{(4-7h_0)\,\xi}{\left(-h_0+2h_0^2\right)h_0} \right\}. \tag{13}$$

For example, we consider a minimal coupling case, $\xi = 0$, here. Since ϕ_0^2 should be positive, one finds $h_0 \leqslant 9/8$.

If we choose $h_0 = -1/60$, it gives the state equation parameter w as

$$w = -1 + \frac{2}{3h_0} = -1.025. \tag{14}$$

It is consistent with the observed one. Therefore, with the proper choice of parameters we obtain the solution for the Einstein equation and the field equation.

As is clearly seen, the Hubble rate H is suppressed as time runs. If we substitute the present age of the universe 10^{10} year into t or $t_s - t$, the observed value of H could be reproduced. This explains the smallness of the effective cosmological constant $\Lambda \sim H^2$. Then by properly choosing the parameters, we may obtain an exact solution for the cosmological constant.

5. Concluding remarks

We have investigated the radiative symmetry breaking and discuss the dynamical resolution of the cosmological constant problem in the scalar self-interacting theory non-linearly coupled with some power of the curvature. We numerically evaluate the one-loop effective Lagrangian in the four-dimensional FRW spacetime with flat spatial part at the weak curvature limit. The phase structure strongly depends on the sign of α. The ξ dependence of it is very weak. For a non-negative α ($\geqslant 0$) we observed the first-order phase transition. Compared with the usual ϕ^4 theory in weakly curved space, i.e. $\alpha = 0$, the expectation value $\langle\phi\rangle$ is extremely enhanced for a positive α. In the case of a negative α the radiative correction is suppressed as curvature decreases. No radiative symmetry breaking is observed for $\alpha < 0$. We apply the mechanism proposed in [10] to our model with negative α. Dynamical mechanism to solve the cosmological constant problem is naturally realized also for the class of models investigated in [1].

As the simplest model, we consider radiative symmetry breaking of a discrete Z_2 symmetry. Breaking of the discrete symmetry must construct a domain wall structure in our universe. It is only a prototype model of the dark energy. We do not expect to explain the entire problem in this simplest model. It is straightforward to perform the same analysis in a complex scalar theory which has a continuous $U(1)$ symmetry. Then we can avoid the domain wall problem and find the same behaviours.

There are many directions where our approach may be generalized. In particular, it is known that the phase structure in the NJL-like model in curved spacetime is quite rich (for a review, see [11]). It is quite interesting to consider models with fermions and gauge bosons. From another point of view, the new matter–gravity coupling [2] may also be considered as a kind of modification of gravitation itself. It should be interesting to calculate the one-loop effective action when the extreme gravity is formulated in the Palatini form. That attracts us to further research works continuously.

Acknowledgments

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